PHY 131 Fall 2018

Test 11: *Thermal Expansion and Conduction*

NAME ______________________________

Thermal Expansion

$\alpha_{steel} = 12 \times 10^{-6}$ 1/°C

$\alpha_{aluminum} = 23 \times 10^{-6}$ 1/°C

$\beta_{water} = 2.1 \times 10^{-4}$ 1/°C

$\Delta L = L_0 \alpha \Delta T$

$\Delta V = V_o \beta \Delta T$ (*liquid*)

Thermal Conductivity

$k_{lath\&plaster} = 0.28$ W/mK

$k_{plaster} = 0.48$ W/mK

$$\frac{\Delta Q}{\Delta t} = \frac{kA\Delta T}{L} \quad \text{(single layer)}$$

$$\frac{\Delta Q}{\Delta t} = \frac{A\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \cdots} \quad \text{(multi − layer)}$$

1 watt = 1 J/sec

25.4 mm = 2.54 cm = 1 in

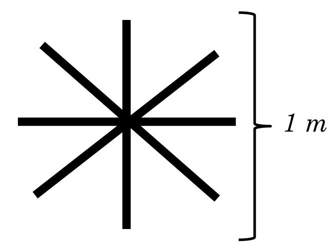
$Vol_{cylinder} = \pi r^2 h$

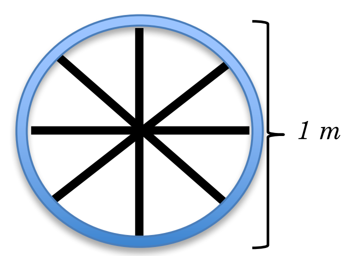
$Circumference = 2\pi r$

$A_{circle} = \pi r^2$

SHOW ALL WORK & PUT YOUR ANSWERS IN THE BOXES PROVIDED

1. (30 points) A blacksmith is asked to make a circular wheel from a strip of steel enclosing wooden spokes. Each spoke has a length of 0.500 meters, so the diameter of the wheel is 1.000 meter. The original length of the piece of steel is 3.115 meters at 20.00 degrees Celsius. To what temperature should the blacksmith heat the steel so that he can wrap it snugly around the wooden spokes as shown in the second diagram? Ignore overlap of steel.





In order to form a circle of diameter 1.000 meters, the steel strip must have a length of 1.000 m × π = 3.1416 m.

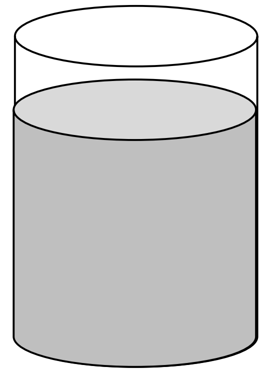
The required length change is thus 3.1416 m - 3.115 m = 0.02659 m. Using the equation gives

$$\Delta L = 0.02659 \text{ m} = L_0 \alpha \Delta T = (3.115 \text{ m})(12 \times 10^{-6}/°\text{C})\Delta T$$

Since ΔT is 711°C, and the steel starts at 20°C, the Blacksmith has to heat the steel to 731°C

731°C

2. (35 points) An aluminum can is filled ¾ full of soda at an equilibrium temperature of 3.0°C. The interior diameter of the can is 2.60 inches, and the interior height is 4.84 inches. (To avoid some math, we are ignoring the narrowing at the base and the lid, treating the can as a right circular cylinder.) When the can of soda is left out in the hot sun, the whole system reaches a temperature of 40°C. What is the change in height of the level of pop in the can? Give your answer in millimeters to two significant digits and indicate whether the level goes up or down. (*Since soda's primary constituent is water, you can use the thermal expansion coefficient for water*.)



When going from 3°C to 40°C, the change in the diameter of the can is

$$\Delta L = L_0 \alpha \Delta T = (2.60\text{ in})(23 \times 10^{-6}/°\text{C})(37°\text{C}) = 0.002213\text{ in}$$

The new diameter is

$$2.60\text{ in} + 0.002213\text{ in} = 2.602213\text{ in}$$

and the new radius is

$$\frac{(2.602213\text{ in})}{2} = 1.301107$$

The original volume of the soda is

$$\text{V} = \pi(1.30\text{ in})^2\,(0.75 \times 4.84\text{ in}) = 19.273\text{ in}^3$$

When going from 3°C to 40°C, the change in the volume of the soda is

$$\Delta V = V\beta\Delta T = (19.273\text{ in}^3)(2.1 \times 10^{-4}/°\text{C})(37°\text{C}) = 0.14975\text{ in}^3$$

The new volume of the soda is

$$19.273\text{ in}^3 + 0.14975\text{ in}^3 = 19.4225\text{ in}^3$$

At the new radius, the level of the soda in the can is

$$\text{V} = 19.4225\text{ in}^3 = \pi(1.301107\text{ in})^2\,h$$

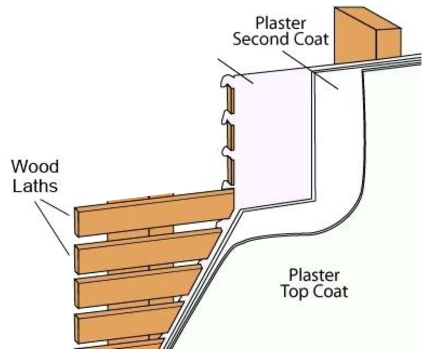
$$h = 3.652\text{ inches}$$

The original height was 3.63 inches, so the change in height is DOWN by 0.022 inches. This is

$$0.022\text{ inches} \times \frac{25.4\text{ mm}}{1\text{ inch}} = 0558\text{ mm}$$

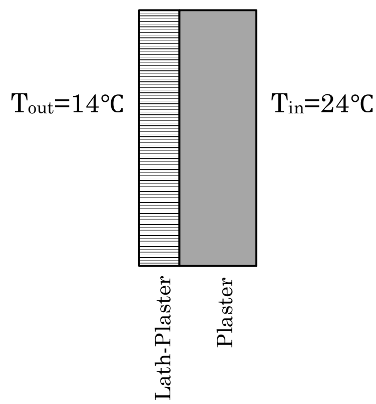
0.56 mm down

3. (35 points) The walls of most modern houses are constructed using 2x4s with insulation between the studs. An older method is lath and plaster: thin strips of wood, lath, were stacked closely with plaster between the strips. Lath and plaster was adapted from an even older method called waddle and dub (a dung-mud mixture between the pieces of wood). Waddle and dub is also known as pine and pug, mud and stud, and my favorite—rab and dab.



Consider a 15-foot by 10-foot wall of the following layers: a 0.25-inch thick lath-and-plaster layer covered by a 0.75-inch plaster layer (comprised of 3 coats), as shown. The temperature inside, on the plaster side of the wall, is 24 degrees Celsius, and the temperature outside, on the lath side of the wall, is 14 degrees Celsius.

a) What is the rate of thermal energy transfer through the wall?
b) What is the temperature at the boundary of the plaster layer and lath-and-plaster layer?



Let's call the temperature of the interface T_i. Since the thermal energy flow rate through the lath-plaster has to be the same as the thermal energy flow rate through the plaster, we have

$$\frac{\Delta Q}{\Delta t} = \frac{(0.28\,\mathrm{W/mK})A(T_i - 14°\mathrm{C})}{0.25''} = \frac{(0.48\,\mathrm{W/mK})A(24°\mathrm{C} - T_i)}{0.75''}$$

Multiplying both sides by 0.75 " and Algebrayifying...

$$0.84\,T_i - 11.76°\mathrm{C} = 11.52°\mathrm{C} - 0.48\,T_i$$

$$1.32\;T_i = 23.28°\mathrm{C}$$

$$T_i = 17.64°\mathrm{C}$$

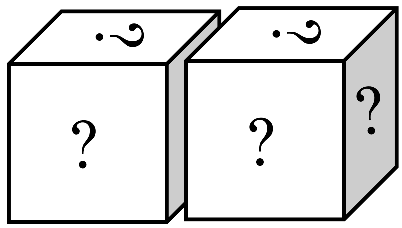
Now part a)

$$\frac{\Delta Q}{\Delta t} = \frac{(0.28\,\mathrm{W/mK})\left(\frac{1\mathrm{m}}{3.28\,\mathrm{ft}}\right)(150\,\mathrm{ft}^2)(17.64°\mathrm{C} - 14°\mathrm{C})}{0.25''\times(1\,\mathrm{ft}/12'')} = 2240\ \mathrm{Watts}$$

2240 Watts
17.6 °C

Bonus (+3):

There is a set of two cubes on which are printed numbers such that each date of the month can be displayed (1 through 31). Single-digit dates are represented with a leading zero (e.g., the first of the month is "01"). What numbers should be printed on each of the 6 sides of each die so that every date of the month can be displayed?



CUBE 1	CUBE 2
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