PHY 131 Fall 2018

Test #6: Work & Energy

Name________________________________

$KE = \frac{1}{2}mv^2$ $EPE = \frac{1}{2}kx^2$ $GPE = mgh$ $W = Fd\cos\Phi$

$a_{cent} = \frac{v^2}{r}$ $\sum F = ma$ $F_{fr,k} = \mu_k F_N$ $F_{fr,s} \leq \mu_s F_N$

$x = \frac{1}{2}at^2 + v_i t$ $x = v_{avg}t$ $v_f^2 = v_i^2 + 2ax$ $v_f = v_i + at$

1609 meters = 1 mile 5280 feet = 1 mile 1.094 yards = 1 meter 3.28 feet = 1 meter

1000 meters = 1 kilometer 60 seconds = 1 minute 60 minutes = 1 hour 1000 grams = 1 kg

Use $g = 10$ m/s^2 = 32 ft/s^2.

SHOW YOUR WORK and PUT YOUR ANSWER IN THE BOX!

1. (30 points) Bryan throws a 200-gram ball straight up in the air with an initial speed of 18 m/s.
a) During the ball's ascent to its maximum height, there are 4.0 J of thermal energy generated by air drag. What is the maximum height reached by the ball above where it was released?
b) The ball returns to Bryan with a speed of 16.0 m/sec. How much thermal energy was generated by air drag during the ball's decent from its maximum height?

a) The initial kinetic energy of the ball is

$$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.2\text{ kg})(18\text{ m/sec})^2 = 32.4\text{ J}$$

It is given that 4.0 J of this is lost to thermal energy during the upward flight. This leaves 28.4 J for gravitational potential energy at its peak. Thus we can calculate the maximum height.

$$28.4\text{ J} = mgh = (0.2\text{ kg})(10\text{ m/sec}^2)h$$

$$h = 14.2\ m$$

b) When the ball returns to Bryan, it has a speed of 16 m/sec and thus a kinetic energy of

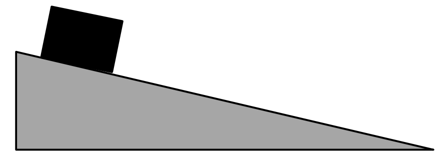
$$KE_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(0.2\text{ kg})(16\text{ m/sec})^2 = 25.6\text{ J}$$

There was 28.4 J of gravitational potential at the peak and only 25.6 J of kinetic at the bottom. Therefore, there must have been 28.4 J -25.6 J = 2.8 J of thermal energy generated during the descent.

a) 14.2 m

b) 2.8 J

2. (30 points) A UPS worker gives a 10.0-kg box a push down an icy driveway. The coefficient of kinetic friction between the box and the ice is 0.300. The driveway is declined at 12.0 degrees below the horizontal. The box slides 1.50 meters before stopping.



a) What is the total thermal energy generated by frictional forces during this trip?

b) What is the initial speed of the box (immediately after his push)?

a) The thermal energy generated by the frictional force is

$$Th\,E = F_{fr}x = \mu_k F_N x = \mu_k mg\cos\theta\ x = (0.300)(10\text{ kg})(10\text{ m/sec}^2)(\cos 12°)(1.5\text{ m}) = 44.0\text{ J}$$

b) The work done be gravity when the block moves 1.5 meters down the incline is

$$W_{grav} = mg\,\Delta h = (10\text{ kg})(10\text{ m/sec}^2)\,(1.5\text{ m}\sin 12°) = +31.2\,J$$

Using the work-energy theorem, we would say that the work done by the frictional force is minus 44.0 J and the work done by the gravitational force is +31.2 J. Therefore, the total work done on the package is minus 12.8 J. Since the final speed of the package is zero, it must have had 12.8 J of kinetic energy to start. Now we can calculate the initial speed.

$$KE_i = 12.8\,J = \frac{1}{2}mv_i^2 = \frac{1}{2}(10\text{ kg})v_i^2$$

$$v_i = 1.60\text{ m/sec}$$

Using the principle conservation of energy, we would say that the total energy in the system initially is due to the gravitational potential energy of the block and the kinetic energy of the block.

$$E_i = \frac{1}{2}mv_i^2 + mgh = \frac{1}{2}(10\text{ kg})v_i^2 + (10\text{ kg})(10\text{ m/sec}^2)(1.5\text{ m}\sin 12°)$$

The expression for the final energy is all thermal.

$$E_f = F_{fr}x = \mu_k F_N x = \mu_k mg\cos\theta\ x = (0.300)(10\text{ kg})(10\text{ m/sec}^2)(\cos 12°)(1.5\text{ m}) = 44.0\text{ J}$$

Setting the initial energy equal to the final energy,

$$\frac{1}{2}(10\text{ kg})v_i^2 + 31.2\text{ J} = 44.0\text{ J}$$

Now we can calculate the initial speed.

$$KE_i = 12.8\,J = \frac{1}{2}(10\text{ kg})v_i^2$$

$$v_i = 1.60\text{ m/sec}$$

The result, of course, is the same.

a) 44.0 J
b) 1.60 m/sec

3. (40 points) Daredevil Vannessa goes bungee jumping! She steps ($v_i = 0$) off of a bridge that is 80 meters above the surface of a river. The relaxed length of the bungee cord is 35 meters. Ignore air drag and treat her as a point mass of 70 kg (which includes her helmet and other protective gear).

a) If she stops *just* above the surface of the river, what is the stiffness constant of the cord?
b) What is her acceleration after she has fallen 50 meters?
c) What is her maximum speed?
d) Complete the table below for this scenario using the river as the level of zero gravitational potential energy. Feel free to use the back of this sheet for calculations.

a) Using the river as the level at which GPE = 0, the initial snapshot is when Vanessa is at rest at the top of the bridge.

$$E_1 = mgh = (70\text{ kg})(10\text{ m/sec}^2)(80\text{ m}) = 56{,}000\,J$$

The second snapshot is when she is at rest (for an instant!) at the river. Here, all of the energy in the system is in the elastic potential of the cord, which is stretched 45 m.

$$E_2 = \frac{1}{2}kx^2 = \frac{1}{2}k(45\text{ m})^2 = 56{,}000\,J$$

Solving for k gives

$$k = 55.3\text{ N/m}$$

b) The two forces on her are mg = 700 N down and kx = (55.3 N/m)(15 m) = 830 N upward. The net force is 700 N - 830 N = 130 N upward. Her acceleration is therefore 130 N/70 kg = 1.86 m/sec^2 upward.

c) Her maximum speed is when the net force changes from downward to upward. That is, when the net force on her is zero. This occurs when $mg = kx$. $x = mg/k$ $=(70\text{ kg})\frac{\left(10\frac{\text{m}}{\text{sec}^2}\right)}{55.3\text{ N/m}} = 12.7\text{ m}$. At this point, the elastic potential is $E_2 = \frac{1}{2}kx^2 = \frac{1}{2}\left(55.3\frac{N}{m}\right)(12.7\text{ m})^2 = 4430$ J and the gravitational potential is $mgh = (70\text{ kg})(10\text{ m/sec}^2)(80\text{ m} - 12.7\text{ m} - 35\text{ m}) = (70\text{ kg})(10\text{ m/sec}^2)(32.3\text{ m}) = 22{,}600\,J$. The energy that is left for kinetic is 56,000 J - 22,600 J - 4430 J = 29,000 J

The maximum speed can be calculated from the maximum KE.

$$29{,}000\text{ J} = \frac{1}{2}(70\text{ kg})v_{max}^2$$

$$v_{max} = 28.8\text{ m/sec}$$

a) 55.3 N/m
b) 1.86 m/sec^2 upward
c) $v_{max} = 28.8\text{ m/sec}$

d)	**EPE** (J)	**GPE** (J)	**KE** (J)	**TOTAL E** (J)
Initial position on the bridge	**0**	**56,000**	**0**	**56,000**
When she has dropped 35 meters	**0**	**31,500**	**24,500**	**56,000**
When she has dropped 50 meters	**6200**	**21,000**	**28,800**	**56,000**
When she is at her lowest point (surface of river)	**56,000**	**0**	**0**	**56,000**

FIVE POINT BONUS

A bungee cord has a length of 30 meters and stiffness constant of 30 N/m. It is cut in half. What is the stiffness constant of half the cord?

a) 15 N/m

b) 30 N/m

c) 60 N/m