PHY 131 Fall 2018

Test #3: Two-dimensional motion

(100 points) Name________________________________

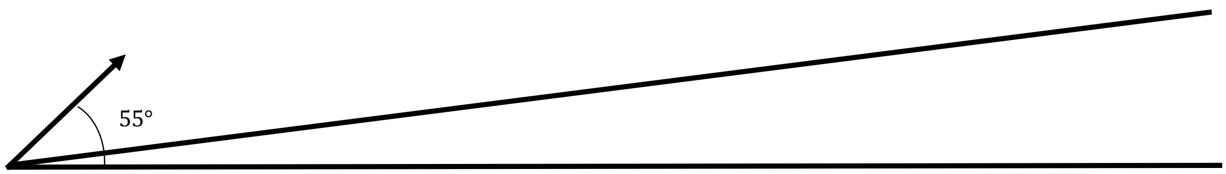
$x = \frac{1}{2}at^2 + v_i t$ $x = v_{avg}t$ $v_f^2 = v_i^2 + 2ax$ $v_f = v_i + at$

1609 meters = 1 mile 5280 feet = 1 mile 3.28 feet = 1 meter *g = -10 m/s² = -32 ft/s².*

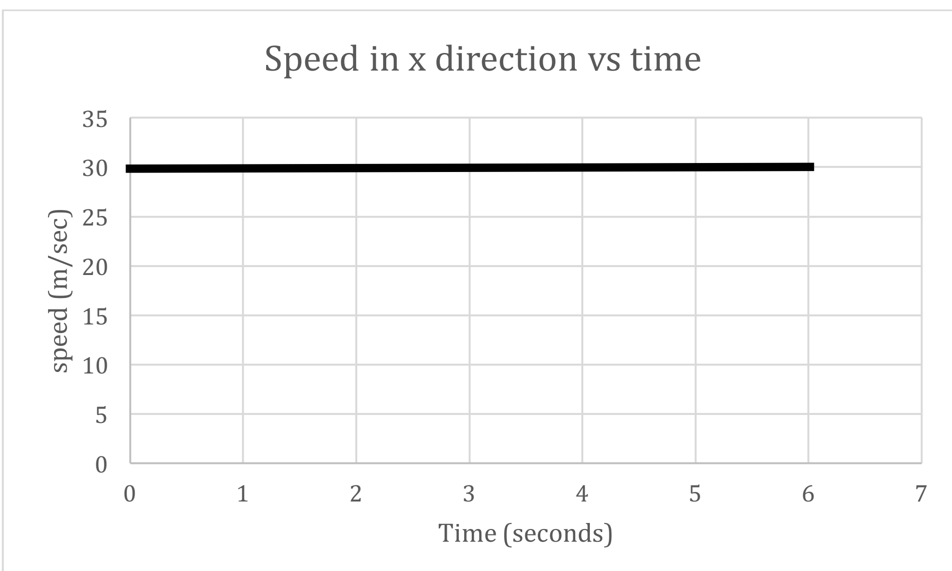
Ignore friction and air resistance for each of these problems.

SHOW YOUR WORK and PUT YOUR ANSWER IN THE BOX!

1. A cannonball is shot into the air from ground level at an angle of 55 degrees above the horizontal up a hill as shown in the diagram. The softball hits the hill 6.00 seconds after it was shot.



A plot of the speed of the cannonball in the horizontal direction, v_x versus t is shown below.



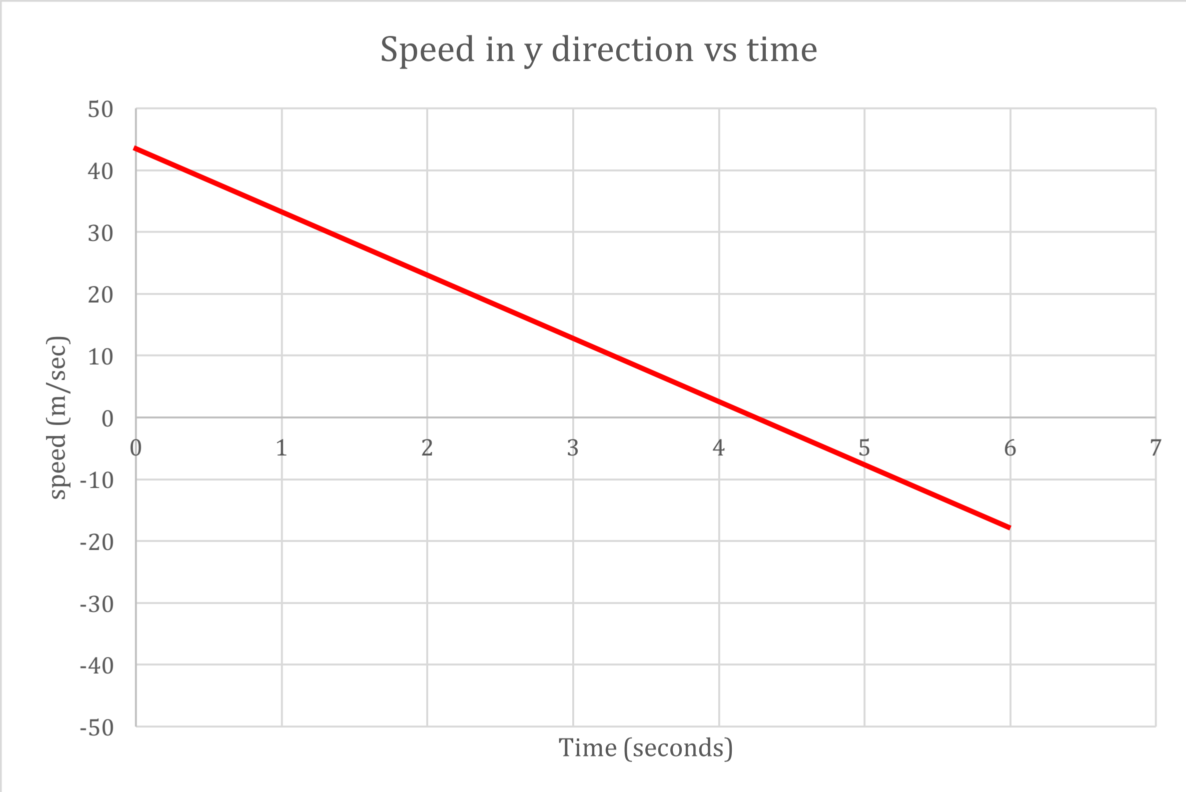
Note: boxes for answers to parts a, b and c and on page 3.

a) What is the initial speed of the cannonball?

b) Defining the origin of the coordinate system (0 m, 0 m) as the point at which the cannonball was shot, determine the (x, y) coordinate of the point at which the cannonball strikes the hill.

c) What is the angle of the hill relative to the horizontal?

d) On the graph provided, plot the vertical of the component of the speed of the cannonball for the entire four seconds of the flight.



a) If v_x is 30 m/second and the angle is 55° relative to the horizontal, the initial speed of the cannonball is

$$v_i = \frac{30\ \mathrm{m/sec}}{\cos 55°} = 52.3\ \mathrm{m/sec}$$

b) The x coordinate of the cannonball at 4 seconds must be 120 meters because it is traveling at a constant speed of 30 m/sec in the horizontal direction. The initial vertical component of the velocity is

$$v_{iy} = (52.3\ \mathrm{m/sec}) \sin 55° = 42.8\ \mathrm{m/sec}$$

With v_y we can determine the y coordinate a number of ways. One is to use the average speed. After six seconds,

$$v_y = 42.8 \frac{\mathrm{m}}{\mathrm{sec}} + \left(-10\ \frac{\mathrm{m}}{\mathrm{sec}^2}\right) 6\ \mathrm{seconds} = -17.2\ \mathrm{m/sec}$$

The average speed in the vertical direction is thus

$$v_y = \frac{42.8 \frac{\mathrm{m}}{\mathrm{sec}} + (-17.2\ \mathrm{m/sec})}{2} = 12.8\ \mathrm{m/sec}$$

The vertical displacement is thus

$$y = (12.8\ \text{m/sec})\ 6\ \text{seconds} = 77.1\ \text{meters}$$

The answer is (120 meters, 77.1 meters)

c) To get the angle of the hill we can use the arctan function

$$\theta = \arctan\frac{77.1}{120} = 32.7°$$

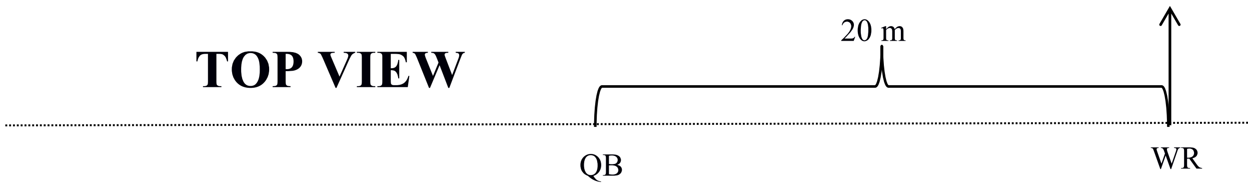
d) see graph.

1 a) 52.3 m/sec

1 b) (120 meters, 77.1 meters)

1 c) 32.7°

2. (50 points) In Canadian Football, the receiver can be running forward at the snap of the ball. The wide receiver is 20 meters from the quarterback at the snap, which we'll define as $t = 0$. (see figure directly below)



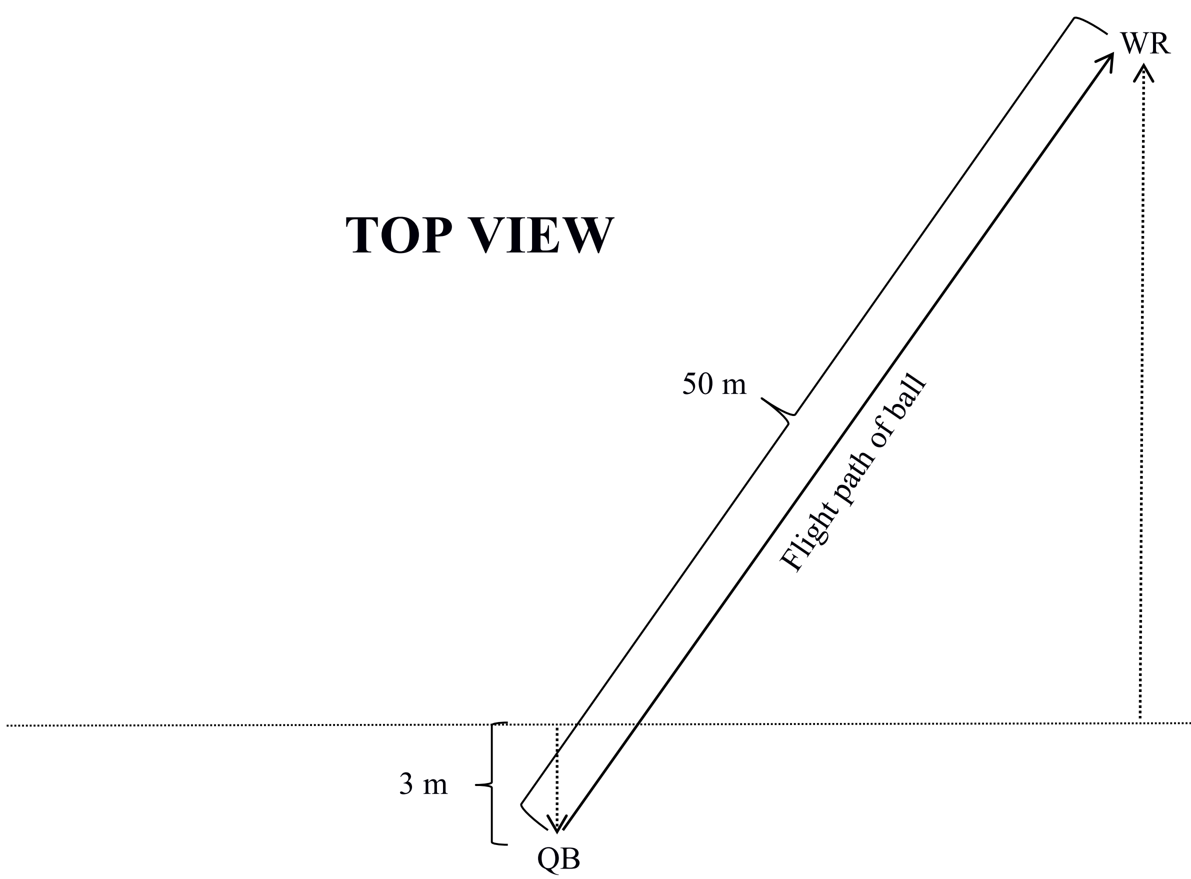
At the snap, the receiver is moving straight down the field at constant speed v_r, and the quarterback drops straight back a distance of 3.00 m and releases the ball at $t = 2.50$ seconds. He throws the ball a distance of 50.0 meters where it is caught by the wide receiver at $t = 4.70$ seconds. Ignore air drag and assume that the ball is caught at the same height from which it was thrown. See figure below depicting an overhead view of the play.

a) What is the speed at which the football was thrown?

b) What is the angle at which the football was thrown a measured from the horizontal (parallel to the ground)?

c) What is the speed of the wide receiver, v_r?

d) Imagine that Kyle Kelly is coming on a safety blitz and the quarterback has to release the ball at $t = 2.00$ seconds instead of 2.50 seconds. If the receiver catches the ball at the same place and the same time, what adjustment did the quarterback make to the **angle** with respect to the horizontal and the **speed** of the ball? Give your answers for both the change in angle and the change in speed to 2 significant figures.



a) The football is in the air for 2.2 seconds and the vertical displacement is zero. The time to the peak is 1.1 seconds, so the initial speed of the football in the vertical direction must be 11 m/sec.

The football travels 50 meters in 2.2 seconds in the horizontal direction, so it must have an initial horizontal speed

$$v_{ix} = \frac{50 \text{ meters}}{2.2 \text{ seconds}} = 22.7 \text{ m/sec}$$

The initial speed of the football is thus

$$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{(22.7 \text{ m/sec})^2 + (11 \text{ m/sec})^2} = 25.2 \text{ m/sec}$$

b)

$$\theta = \arctan\frac{11}{22.7} = 25.9°$$

c) To get the speed of the wide receiver, we need to know the distance the wide receiver travels and the time taken. The time is simply 4.7 seconds, which is the time between the snap and the receiver catching the ball.

Using the Pythagorean theorem, we can calculate the distance upfield between the QB and the WR

$$y = \sqrt{(50 \text{ m})^2 - (20 \text{ m})^2} = 45.8 \text{ m}$$

Subtracting 3 m for the distance the QB dropped behind the line of scrimmage, we get 42.8 meters for the distance the WR ran. His speed is thus

$$v_r = \frac{42.8 \text{ m}}{4.7 \text{ seconds}} = 9.11 \text{ m/sec}$$

d) Now the ball has to be in the air for 2.7 seconds instead of 2.2 seconds. If the ball is in the air for 2.7 seconds, it reaches the peak in 1.35 seconds, so the initial speed in the vertical direction must be 13.5 m/sec.

In the horizontal direction, the ball travels 50 m in 2.7 seconds, which means it has a horizontal speed of

$$v_{ix} = \frac{50 \text{ meters}}{2.7 \text{ seconds}} = 18.5 \text{ m/sec}$$

The initial speed of the football is thus

$$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{(18.5 \text{ m/sec})^2 + (13.5 \text{ m/sec})^2} = 22.9 \text{ m/sec}$$

and the angle above the horizontal is:

$$\theta = \arctan\frac{13.5}{18.5} = 36.1°$$

The adjustment is thus 25.2 m/sec - 22.9 m/sec = 2.3 m/sec slower and 36.1° − 25.9°= 10.2° higher.

2 a) 25.2 m/sec

2 b) 25.9°

2 c) 9.11 m/sec

d) He threw the ball ___2.3___ m/sec ___slower___ at an angle of ___10___ degrees ___higher___.
(*faster* or *slower*) (*higher* or *lower*)