PHY 131 Fall 2018

Test #9: *Simple Harmonic Motion*

NAME ________________________________

*****Remember to have your calculator mode in radians!*****

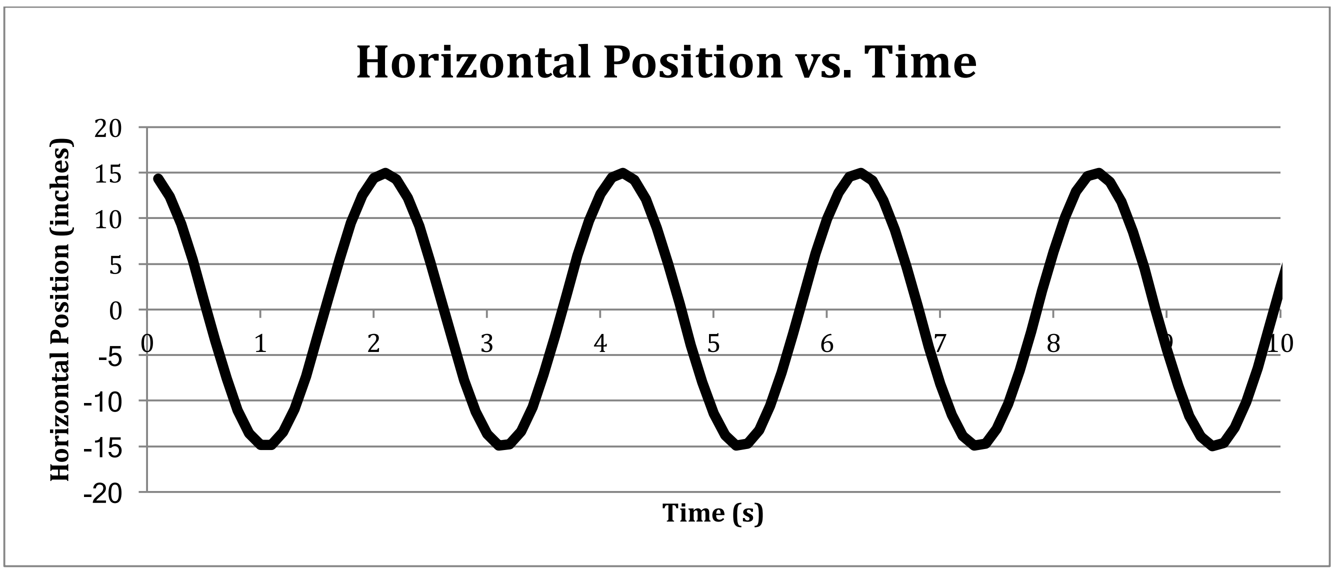
$$\text{If } x(t) = A\cos\omega t \text{ then } v(t) = -A\omega\sin\omega t$$

$$\omega = \frac{2\pi \text{ radians}}{\text{cycle}} f$$

$$T = \frac{1}{f}$$

SHOW ALL WORK and PUT YOUR ANSWERS IN THE BOX

1. (30 points) Consider the graph for a pendulum released from rest at time $t = 0$ seconds for the following questions:
   a) What is the period of the motion?
   b) What is the amplitude of the motion?
   c) How many times does the pendulum bob swing through its lowest point in the time from $t = 0$ to 8 seconds?



a) $\frac{2\pi}{3}$ seconds $= 2.1$ seconds

b) 15 cm

c) 8 times

2. (30 points) A rock is stuck in the tread of a tire that is 20 inches in diameter. The tire makes 40 rotations per minute.

a) What is the oscillation frequency of the rock in units of Hertz?
b) Write the equation for the vertical position of the rock in time, y(t), where y=0 at the pavement. Define $t = 0$ when the rock is touching the pavement.
c) What is the maximum vertical speed of the rock in units of inches/second?

b) If the frequency is 40 RPM, the tire makes two-thirds of a revolution per second. The angular frequency is two-thirds of 2π radians/second = $4\pi/3$ radians /second = 4.19 rad/sec.

The amplitude is 10 inches, and general shape of the curve is a negative cosine. HOWEVER, the cosine equation will range from minus 10 inches to plus 10 inches and the height of the rock varies from 0 to 20 inches. To make this adjustment, the entire curve must be shifted up ten inches.

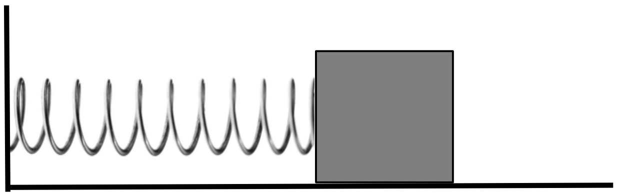
$$y(t) = -10 \text{ inches} \cos\left(4.19 \frac{\text{rad}}{\text{sec}}\, t\right) + 10 \text{ inches}$$

c) The maximum vertical speed of the rock is only a function of the rotation rate of the tire and not the speed of the car itself. If the tire makes 40 rotations per minute, the rock has a speed of

$$v = \frac{x}{t} = \frac{40[2\pi(10 \text{ inches})]}{60 \text{ seconds}} = 41.9 \text{ in/sec}$$

a) 0.67 Hz
b) $y(t) = -10 \text{ inches} \cos\left(4.19 \frac{\text{rad}}{\text{sec}}\, t\right) + 10 \text{ inches}$
c) 42 in/sec

3. (40 points) A spring is compressed by pushing a 2.00-kg mass attached to the end of it, as shown. The mass is released at $t = 0$, and then slides back and forth along the frictionless surface. Its maximum speed is 30.0 cm/sec. The stiffness of the spring is 65.0 N/m and the period of the motion is 1.10 seconds.



a) What is the equation that gives the horizontal position of the mass as a function of time? (include appropriate units.)
b) On the axes provided on the next page, neatly plot the position vs. time for 3 complete cycles of this motion. Label your values for **amplitude** and **times for midpoints and endpoints** of the motion.
c) If the surface were not frictionless, the oscillations would be damped and the mass would eventually come to a stop. Let's say it comes to a stop with the spring at its relaxed length. At this point, how much work would friction have done on the mass? (remember to include the proper sign on the work done by friction)

a) To get the amplitude, we use conservation of energy. The maximum kinetic energy is

$$KE_{max} = \frac{1}{2}(2\text{ kg})(0.30\text{ m/sec})^2 = 0.09\text{ J}$$

This must also be the maximum energy that is stored in the elastic potential of the spring.

$$0.09\text{ J} = \frac{1}{2}kx^2 = \frac{1}{2}\left(65\ \frac{\text{N}}{\text{m}}\right)(x_{max})^2$$

Solving gives, x_{max}= 16.6 cm. If the period is 1.10 seconds, the frequency is 0.91 Hz, which is 5.71 radians per second. Therefore, the equation of motion is:

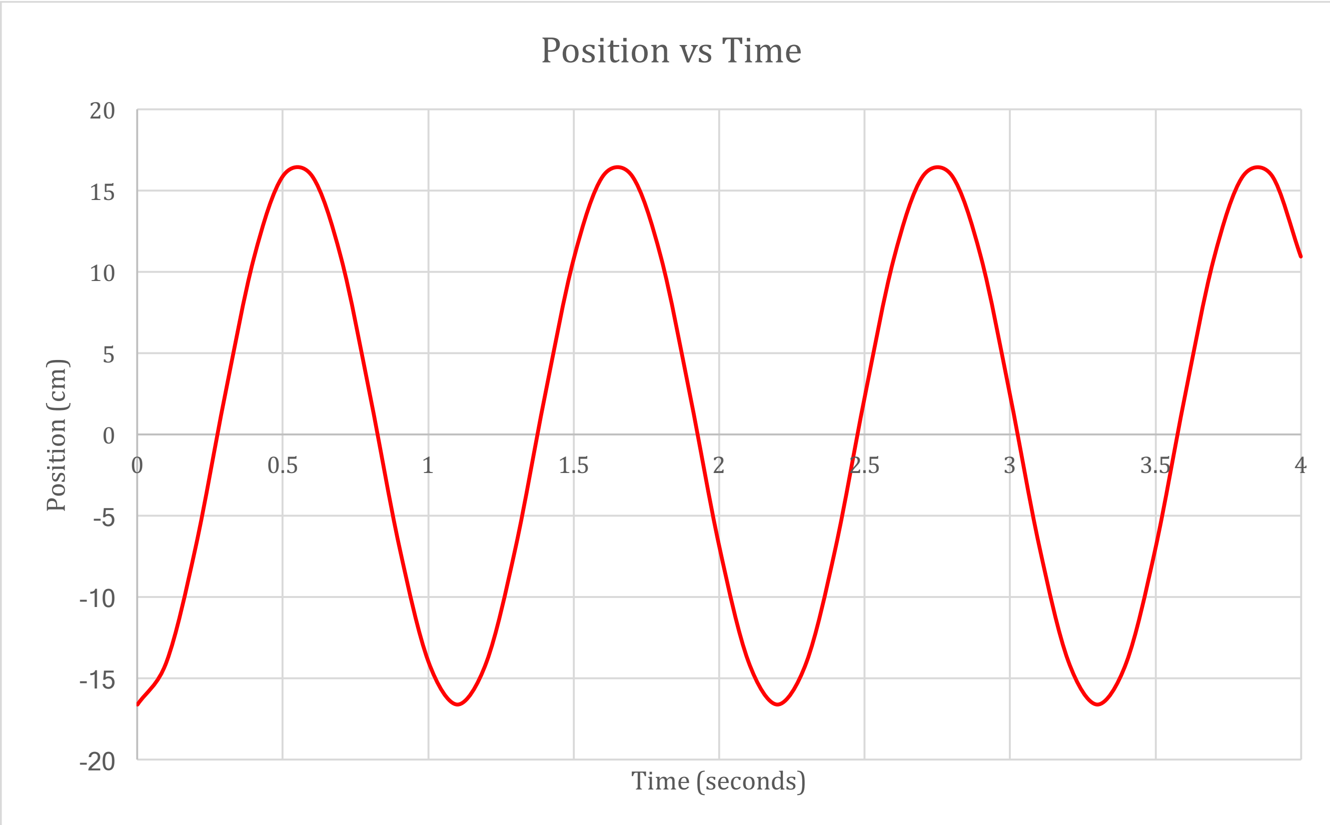
$$x(t) = -16.6\text{ cm}\cos\left(5.71\ \frac{\text{radians}}{\text{second}}\,t\right)$$

b) see graph

c) All of the 0.09 J of energy in the system is now thermal.

a) $x(t) = -16.6\text{ cm}\cos\left(5.71\ \frac{\text{radians}}{\text{second}}\,t\right)$

c) Must be -0.09 J



Statics Make-Up - 80 points possible
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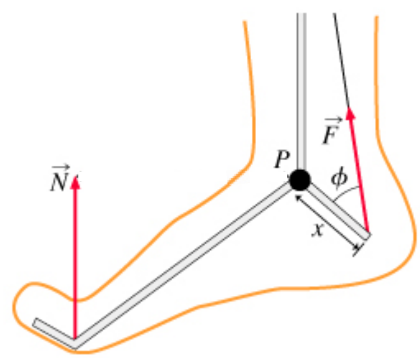
A 70 kg person stands on the ball of one foot as shown in the diagram. The ankle joint is modeled with four bones and the Achilles’ tendon as shown. We are going to call these the toe bone, foot bone, heel bone, and leg bone. The foot bone is 8” long and the heel bone is 2.5” long. If the angle between the foot bone and the floor is 30° and the angle between the Achilles’ tendon and the heel bone is 40°, what is the tension in the Achilles’ tendon?



Taking torques about the point P, which represents the ankle joint, we have

$$\tau_{cw} = (8")F_N \sin 60° = (8")(700\text{ N}) \sin 60° =$$

$$\tau_{ccw} = (2.5")\, T \sin 40°$$

Setting these equal and solving for T gives:

$$T = \frac{(8")F_N \sin 60°}{(2.5") \sin 40°} = \mathbf{3000\ N}$$