PHY 131 Fall 2018

Test #5: Circular Motion & Forces

(100 points) Name________________________________

$$x = \frac{1}{2}at^2 + v_i t \qquad x = v_{avg}t \qquad v_f^2 = v_i^2 + 2ax \qquad v_f = v_i + at$$

$$F_{net} = ma \qquad F_F = \mu F_N \qquad \sum F_{cent} = \frac{mv^2}{r} \qquad c = 2\pi r$$

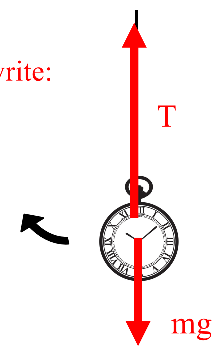
1609 meters = 1 mile 5280 feet = 1 mile 3.28 feet = 1 meter g = 10 m/s^2 = 32 ft/s^2

1000 grams = 1 kg 100 cm = 1 m

Ignore friction and air resistance unless otherwise directed.

SHOW YOUR WORK and PUT YOUR ANSWER IN THE BOX!

1. (30 points) Jesse swings an antique pocket watch around in a *vertical* circle. The chain has a length of 25 cm, and the mass of the watch is 100 grams.
   a) If the watch chain can withstand a maximum tension of 12 newtons, what is the maximum RPM (revolutions per minute) with which he can swing the watch without the chain breaking?
   b) If he swings it too quickly, at what point in the circular path of the watch is the chain most likely to break? Be specific.



The greatest tension will be with the watch is in the bottommost position. At this point, we can write:

$$T - mg = ma_y$$

And we can set T = 12 N and mg = 1.0 N. This makes

$$ma_y = 11\ N = m\frac{v^2}{r} = \frac{(0.1\text{ kg})v_{max}^2}{0.25\text{ m}}$$

Solving for v_{max} gives

$$v_{max} = \sqrt{\frac{(11\text{ N})(0.25\text{ m})}{0.1\text{ kg}}} = 5.244\text{ m/sec}$$

Each revolution is

$$C = 2\pi R = 2\pi(0.25\text{ m}) = 1.57\text{ m}$$

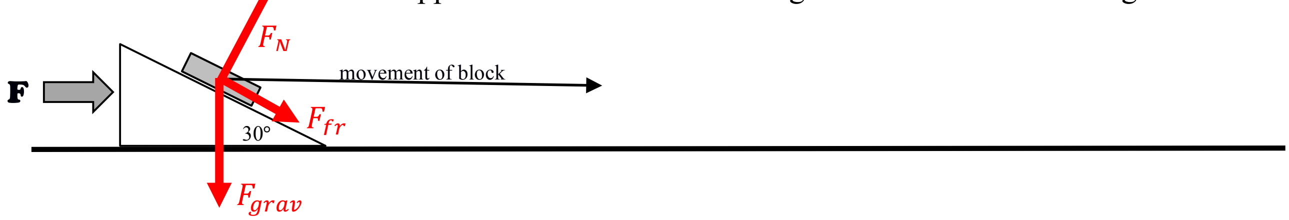
The angular speed is thus

$$5.244\frac{\text{m}}{\text{sec}} \times \frac{1\text{ revolution}}{1.571\text{ m}} \times \frac{60\text{ seconds}}{1\text{ minute}} = 200\text{ RPM}$$

a) 200 RPM

b) When the watch is in its lowest position.

2. (30 points) A wedge on a level track has an angle of (see figure). A block is place on its slanted surface and a horizontal force is applied to the back of the wedge to accelerate it to the right.



a) If there is no friction between the block and the plane, what must be the acceleration of the wedge so the block remains stationary on the wedge? That is, so that the block moves only horizontally.
b) If coefficient of static friction between the block and the wedge is of 0.500 and the coefficient of kinetic friction between the block and the wedge is of 0.350, what is the maximum acceleration of the wedge at which that the block will only move horizontally?

a) With no friction, all the horizontal acceleration is provided by the normal force. Since the acceleration is horizontal, we put the x axis horizontal. We then have

$$F_x = ma_x = F_N \sin 30°$$

in the horizontal direction and

$$F_y = ma_y = 0 = F_N \cos 30° - mg$$

Substituting the expression for F_N from the y direction into the x direction equation we get

$$ma_x = F_N \sin 30° = \left(\frac{mg}{\cos 30°}\right) \sin 30° = mg \tan 30°$$

The acceleration is $g \tan 30°$, which is 5.8 m/sec^2.

b) Now we add in the frictional force.

$$F_x = ma_x = F_N \sin 30° + F_{fr} \cos 30°$$

$$F_y = ma_y = 0 = F_N \cos 30° - mg - F_{fr} \sin 30°$$

We can substitute $\mu_s F_N$ for the frictional force.

$$ma_{x,max} = F_N \sin 30° + \mu_s F_N \cos 30°$$

$$F_N \cos 30° = mg + \mu_s F_N \sin 30°$$

Algebraifying…

$$ma_{x,max} = mg \frac{(\sin 30° + \mu_s \cos 30°)}{(\cos 30° - \mu_s \sin 30°)}; \quad a_{x,max} = g \frac{(\sin 30° + \mu_s \cos 30°)}{(\cos 30° - \mu_s \sin 30°)} = 15.1 \text{ m/sec}^2$$

a) 5.8 m/sec^2
b) 15.1 m/sec^2

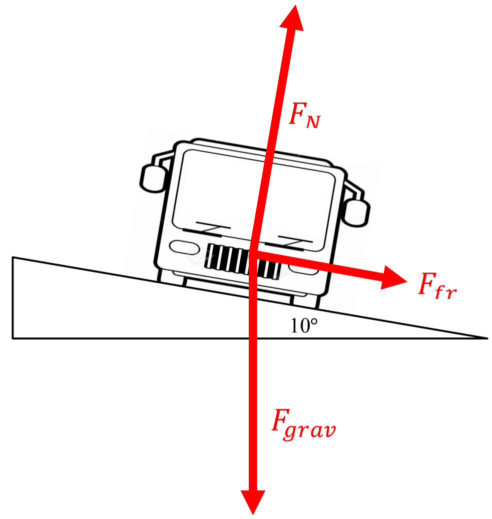
3. (40 points) A 2500-kg truck drives around a curve that is banked inward at 10 degrees above the horizontal. The radius of the curve is 50 meters and the truck is traveling at a constant speed v. The force of friction between the tires of the truck and road is 4000 N.

a) Draw a force diagram on the figure to the right. Include and label all the forces ON the truck, correctly indicating the direction of the frictional force.

b) Defining the x axis as horizontal and to the right as positive, write the sum of the forces in the x direction.

c) Defining the y axis as vertical and up as positive, write the sum of the forces in the y direction.

d) What is the speed of the truck? Express your answer in units of mph to two significant digits.



In the horizontal direction we have

$$F_N \sin 10° + F_{fr} \cos 10° = ma_x = m\frac{v^2}{r}$$

and in the vertical direction we have

$$F_N \cos 10° - F_{fr} \sin 10° - mg = ma_y = 0$$

It is given that F_{fr}= 4000 N. We can solve for F_N using the vertical equation to get

$$F_N = \frac{F_{fr} \sin 10° - mg}{\cos 10°} = \frac{4000 \text{ N} \sin 10° + 25{,}000 \text{ N}}{\cos 10°} = 26{,}091 \text{ N}$$

Substituting this result into the horizontal direction equation.

$$(26091 \text{ N}) \sin 10° + (4000 \text{ N}) \cos 10° = ma_x = (2500 \text{ kg}) \frac{v^2}{(50 \text{ meters})}$$

The result is, $v = 13.0 \text{ m/sec}$

Converting to mph…

$$v = \frac{13.0 \text{ m}}{\text{sec}} \times \frac{1 \text{ mile}}{1609 \text{ m}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} = 29.1 \text{ mph}$$

b) $F_N \sin 10° + F_{fr} \cos 10°$

c) $F_N \cos 10° - F_{fr} \sin 10° - mg$

d) v = 29.1 mph